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A new method of measuring $\frac{\Delta\Gamma}{\Gamma}$ in the $B_s^0 - \bar{B}_s^0$ system

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Abstract. A new way to measure $\frac{\Delta\Gamma}{\Gamma}$ in the $B_s^0 - \bar{B}_s^0$ System based on a comparison of the measured B_s^0 and B_d^0 lifetimes is introduced. This allows to use data from *all* the experiments simultaniously. An upper limit of $\frac{\Delta\Gamma}{\Gamma} \leq 0.24$ at 95 % CL can be given. This is below the theoretical upper limit of 0.27.

1 Introduction

As in the $K^0 - \bar{K}^0$ system, there is a mixing of the flavour states B^0 and $\bar{B}^0$, which results in the new states B_S and B_L with the (probably) different width Γ_S und Γ_L. For B_d^0 it is expected that $\frac{\Delta\Gamma}{\Gamma} \approx 0.0$, while in the $B_s^0 - \bar{B}_s^0$ system $\frac{\Delta\Gamma}{\Gamma} \approx 0.17$ is expected [1]. In this paper, only the $B_s^0 - \bar{B}_s^0$ System is considered.

For a decay into a final state f we set

$$\Gamma_f = \begin{pmatrix} \Gamma_{f,11} & \Gamma_{f,12} \\ \Gamma_{f,21} & \Gamma_{f,22} \end{pmatrix} = \begin{pmatrix} <B^0|f><f|B^0> & <B^0|f><f|\bar{B}^0> \\ <\bar{B}^0|f><f|B^0> & <\bar{B}^0|f><f|\bar{B}^0> \end{pmatrix}$$

Obviously $\Gamma_{f,21} = \Gamma^*_{f,12}$.
If CP-violation is neglected and the initial state is not tagged, the following time-dependence for the decay rates is obtained:

$$\begin{aligned} P_{B_s^0\bar{B}_s^0 \to f}(t) &= \frac{1}{2}(e^{-\Gamma_L t} + e^{-\Gamma_S t})\ (\Gamma_{f,11} + \Gamma_{f,22}) \\ &\quad -(e^{-\Gamma_S t} - e^{-\Gamma_L t})\ \mathrm{Re}\Gamma_{f,12} \\ &= \left(\frac{1}{2}(\Gamma_{f,11} + \Gamma_{f,22}) - \mathrm{Re}\Gamma_{f,12}\right)\ e^{-\Gamma_S t} \\ &\quad + \left(\frac{1}{2}(\Gamma_{f,11} + \Gamma_{f,22}) + \mathrm{Re}\Gamma_{f,12}\right)\ e^{-\Gamma_L t} \\ &= A\ e^{-\Gamma_S t} + B\ e^{-\Gamma_L t} \end{aligned}$$

where $\Gamma = \frac{1}{2}(\Gamma_S + \Gamma_L)$ and A and B are defined as the short- and longlived amplitudes.

Most of the experiments measuring the B_s^0-lifetime do completely neglect $\Delta\Gamma$, and due to that, they only fit with *one* exponential function. This results in the following dependence of the measured lifetime on $\Delta\Gamma$

$$\tau_{B_s^0} = \frac{(A\frac{1}{\Gamma_S^2} + B\frac{1}{\Gamma_L^2})}{(A\frac{1}{\Gamma_S} + B\frac{1}{\Gamma_L})}$$

Four different cases have to be considered :

– CP-modes: In the case of a decay into a CP-eigenstate, either A or B is 0, that means $\Gamma = \Gamma_S$ or $\Gamma = \Gamma_L$. This provides a very easy and clean way to measure $\frac{\Delta\Gamma}{\Gamma}$, but the main drawback is the very low statistics due to the small branching ratios.
– Totally inclusive modes: In these modes $A = \Gamma - \Delta\Gamma$ and $B = \Gamma + \Delta\Gamma$.
– Semileptonic modes: In this case $\Gamma_{f,11} = \Gamma_{f,22} = \Gamma_{sl}$ and $\Gamma_{f,12} = \Gamma_{f,21} = 0$, so A=B and the result simplifies to

$$\tau_{B_s^0} = \frac{1}{\Gamma}\frac{1 + (\frac{\Delta\Gamma}{2\Gamma})^2}{1 - (\frac{\Delta\Gamma}{2\Gamma})^2}$$

For small $\Delta\Gamma$ this leads to

$$\tau_{B_s^0} \approx \frac{1}{\Gamma}\left[1 + 2\left(\frac{\Delta\Gamma}{2\Gamma}\right)^2\right]$$

If $\Delta\Gamma$ increases, the measured lifetime also will. The increase is quadratic to lowest order. This is *only* true for A=B. Theoretical calculations imply that $\Gamma = \Gamma_{B_d^0}$ to an accuracy of about 1% [1]. Using this, $\Delta\Gamma$ can be calculated as a function of the measured B_d^0 and B_s^0 lifetimes

$$\frac{\Delta\Gamma}{\Gamma} = 2\sqrt{\left(\frac{\tau_{B_s^0}}{\tau_{B_d^0}} - 1\right) / \left(1 + \frac{\tau_{B_s^0}}{\tau_{B_d^0}}\right)}$$

Please note that $\tau_{B_s^0}$ should always be *larger* than $\tau_{B_d^0}$.
– Other modes: In the general case, e.g for partially inclusive modes like D_sX, the amplitudes are not known with great precision. This is a problem, because the lifetime-shift is now *linear* in $\frac{\Delta\Gamma}{2\Gamma}$:

$$\tau_{B_s^0} = \frac{(A\frac{1}{\Gamma_S^2} + B\frac{1}{\Gamma_L^2})}{(A\frac{1}{\Gamma_S} + B\frac{1}{\Gamma_L})}$$

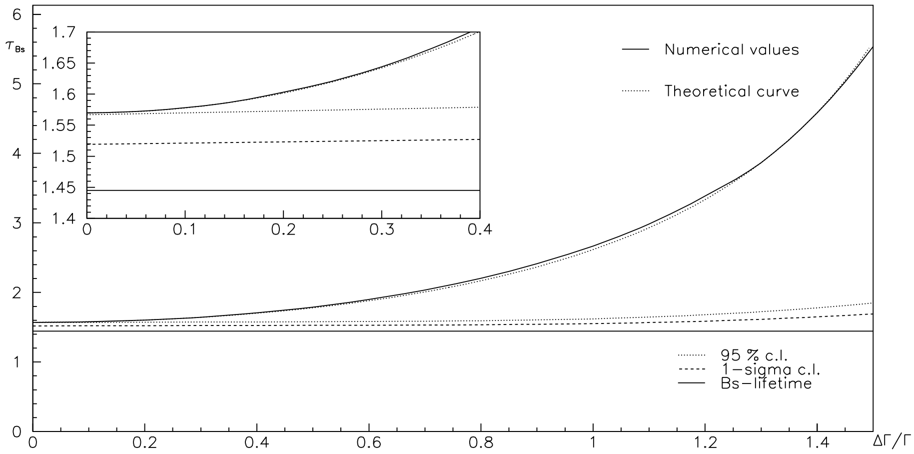

Fig. 1. Expected B_s^0 lifetime as a function of $\frac{\Delta\Gamma}{\Gamma}$ and measured value. The insert shows an enlarged region

$$\approx \frac{1}{\Gamma}\left[1+\frac{A-B}{A+B}\frac{\Delta\Gamma}{2\Gamma} + \left(1+4\frac{AB}{(A+B)^2}\right)\left(\frac{\Delta\Gamma}{2\Gamma}\right)^2\right]$$

If a certain relative uncertainty of the amplitudes is assumed, the relative uncertainty of $\frac{\Delta\Gamma}{\Gamma}$ is at least 5 times as large. If this uncertainty should not be greater then 10 %, the amplitudes must be known at least to 2 % accuracy, which is very difficult.

Due to the problems stated above, we prefer to use only lifetime measurements based on semileptonic decays. The improvement in statistics by using all measurements is very small anyway.

2 Numerical calculation

In a real measurement, the lifetime shift may depend on the background or the detector resolution. Not all those effects can be treated analytically, and so a numerical program has been used to study them. This program computes the log-likelihood-function by numerical integration, including effects like resolution, cut offs and background. The signal-function, representing the data, consist of two exponentials, the fit-function only of one. The program minimizes the log-likelihood for the only free parameter, the B_s^0-lifetime. The computed value is the (expected) measured lifetime depending on $\Delta\Gamma$ or the background size.

The difference between the theoretical formula and the numerical calculation, as well as the systematic error of the numerical calculation depend on the size of $\frac{\Delta\Gamma}{\Gamma}$. Because of this, the difference and the errors are stated in Table 1 seperately for different values of $\frac{\Delta\Gamma}{\Gamma}$.

Table 1. All values are given in fs. The difference is $\tau_{theoretical} - \tau_{numerical}$. * *Important*: This error is *not* added. See text below

Source of error	$\frac{\Delta\Gamma}{\Gamma} \leq 0.5$	$\frac{\Delta\Gamma}{\Gamma} \leq 1.0$	$\frac{\Delta\Gamma}{\Gamma} \leq 1.5$
Lifetime cuts*	1 + 1	17 + 29	290 + 234
Resolution	-2 ± 12	-8 ± 32	-16 ± 49
Background size	-8 ± 3	-60 ± 17	-179 ± 38
Background fit	0 ± 20	0 ± 55	0 ± 213
Total difference ± error	-11 ± 24	-52 ± 66	65 ± 222

– Lifetime cuts: Because every detector has a finite length, only events with smaller decay length are recorded. Here a maximum decay length of 7 cm has been choosen, while all detectors under consideration have decay length larger then 10 cm . If the maximal decay length is short, more long-lived events are cut off, and the fitted lifetime is shifted to a smaller value. This leads to an overestimation of $\Delta\Gamma$, which means that this is the most conservative approach for giving an upper limit on $\frac{\Delta\Gamma}{\Gamma}$. Due to that, the error is not added to the total systematic. It is stated here only for completion.
– Resolution: The resolution was parametrized with two Gaussians, similiar to the resolution functions used at LEP. The two width were varied from 0 to 4 times of the experimental values.
– Background size: Here only the *size* of the background was varied, the values for the generating and the fit function were the same. For the background, a model taken from the LEP experiments was used, consisting of a combinatorical background with positive, negative and zero lifetime, and backgrounds for c- and other b-decays. These parameters, which are very similiar for all experiments, were varied by ±20 % around the mean value to obtain the error.
– Background fit: Another bias is introduced, if the background assumed in the lifetime fit is differrent from the real background. Since we only fit for $\tau_{B_s^0}$, the errors from the background uncertainty have to be introduced artificially. This is done by using different values for the background ratios in the fit- and in the signal-function.
The error of these ratios is given by the experiments. To check the result, we compared the lifetime error for $\frac{\Delta\Gamma}{\Gamma} = 0.0$ with the systematic uncertainty on the lifetime due to the background ratios given by the exper-

iments. Our error is 9 fs on $\tau_{B_s^0}$, while the experiments state 13 fs. To be consistent, the error was rescaled for all values of $\frac{\Delta\Gamma}{\Gamma}$.

The effect of the various parameters is surprisingly small, at least for values of $\frac{\Delta\Gamma}{\Gamma}$ below 0.5. Therefore it is possible to use the results from all experiments, even without knowing all the parameter values in detail.

For the final result, the total systematic error is added in quadrature to the error of the lifetime measurements. The error for $\tau_{B_s^0}$ at $\frac{\Delta\Gamma}{\Gamma} = 0$ is taken to be 13 fs.

If all measurements of the B_s^0-lifetime using semileptonic decays from ALEPH [2], CDF [3], DELPHI [4] and OPAL [5] are combined, $\tau_{B_s^0} = (1445 \pm 65)$ fs is obtained using the averaging method of the LEP-blifetime group [6].

The lifetime of B_d^0 is [8]: $\tau_{B_d^0} = (1570 \pm 30)$ fs. There is an additional error of 16 fs from the theoretical uncertainty of the ratio of Γ and $\Gamma_{B_d^0}$ [1].

The result is shown in Fig. 1:

The errors for the $\tau_{B_d^0}$ and $\tau_{B_s^0}$ measurements (including the theoretical error) were added in quadrature to the systematic errors.

Obviously, there is no intersection point. The reason is, that $\tau_{B_s^0}$ is *lower* than $\tau_{B_d^0}$, while the prediction is the other way round. In a naive interpretation, this means, that a lifetime difference greater than 0 is excluded with a CL of about 96 %. This CL is defined such that only 4 % or less of experiments would result in a B_s^0 lifetime as observed or lower, if $\frac{\Delta\Gamma}{\Gamma}$ was larger than 0.

This might indicate a problem, either of the theoretical prediction that $\Gamma = \Gamma_{B_d^0}$ to an accuracy of about 1% [1] or the experimental data. The theoretical assumption has been checked again, and so there is evidence to believe, that there may be a problem in some of the experimental results.

Ignoring these problems, an upper limit can be given by using a new method [7], which is also recommended by the PDG in such situations. The result is

$$\frac{\Delta\Gamma}{\Gamma} \leq 0.10 \qquad 68\% \text{ CL}$$
$$\frac{\Delta\Gamma}{\Gamma} \leq 0.24 \qquad 95\% \text{ CL}$$

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